One-dimensional simulation of time-dependent behavior of clay

Nagoya Institute of Technology, Associate Professor Member ○ H. M. Shahin
Nagoya Institute of Technology, Professor Member Teruo Nakai
Nagoya Institute of Technology, Researcher Member Mamoru Kikumoto
Nagoya Institute of Technology, Graduate Student Student Member Hiroyuki Kyokawa, Yoshito Miyahara

ABSTRACT

Simulations of one-dimensional consolidation test on clay have been performed using a one-dimensional model which can consider the effects of density, bonding and time. The model can explain the strain rate effect, stress relaxation characteristics and creep characteristics of soils comprehensively without fitting the model for a particular phenomenon. The model can also describe other characteristics of soils such as secondary compression, delayed consolidation and consolidation characteristics of naturally deposited soils. Here, consolidation behavior of clay has been investigated varying the density of the soil having different states of bonding. Varying the thickness of the soil samples, the dependency of the sample height on the consolidation behavior has been checked as well. It is found that the model can explain well the behavior of secondary consolidation of naturally consolidated soil, over consolidated soil and structured soil.

1. INTRODUCTION

Based on the constitutive modeling of geomaterials described in the reference [7] the characteristics of time effect of soils are presented in this paper in a simple and rational way. It can explain the strain rate effect, stress relaxation characteristics and creep characteristics of soils comprehensively without fitting the model for a particular phenomenon. This model is not restricted to normally-consolidated soil; it can explain the characteristics of time effect in over-consolidated and structured soils as well. In this model the behavior of each element of the soil skeleton at every moment in time is determined with void ratio, effective stress, strain rate which are being experienced by the element at that moment. Any origin for the measurement of time is not required in this model for primary and secondary stages.

2. DESCRIPTIONS OF THE MODEL

The framework of the model has been described in reference [7]. Here, a brief description of the model will be given. Fig.1 shows a typical void ratio (e) – stress in logarithmic scale ($\ln\sigma$) relation of normally consolidated (NC) clay. The figure illustrates the change of void ratio for the change of state of stress from point I to P. Here, e_0 and e are the initial and current void ratios of structured soil, and e_{N0} and e_N are the corresponding void ratios on the normally consolidation line. Slopes of the straight lines λ and κ denote the compression and swelling indices. For this condition, the one-dimensional formulation of the model (reference [7]), referring to Fig.1, is

$$d(-e) = d(-e)^p + d(-e)^e = \left\{(\lambda-\kappa)+\kappa\right\}\frac{d\sigma}{\sigma} \qquad (1)$$

Here, $d(-e)$ is the total change of void ratio, superscript 'p' denotes plastic component and 'e' represents elastic component.

Fig.2 shows the change of void ratio when the stress condition moves from the initial state I ($\sigma=\sigma_0$) to the current state P ($\sigma=\sigma$) in overconsolidation condition. The total change of void ratio in over consolidation condition is (reference [6])

$$d(-e) = d(-e)^p + d(-e)^e = \left\{\frac{\lambda-\kappa}{1+G(\rho)}+\kappa\right\}\frac{d\sigma}{\sigma} \qquad (2)$$

Where, ρ is a state variable which represents the density of soils, and $G(\rho)$ is a increasing function of ρ which satisfies $G(0)=0$. The evolution rule of ρ is given by

$$d\rho = -G(\rho)\cdot d(-e)^p \qquad (3)$$

Fig.3 shows the change of void ratio in the case of structured soil when the stress condition moves from the initial state I ($\sigma=\sigma_0$) to the current state P ($\sigma=\sigma$). For the condition of structured soil, the total change of void ratio is (reference [7])

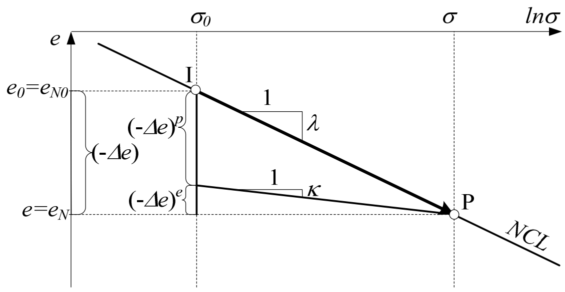

Fig. 1. Change of void ratio in NC clay

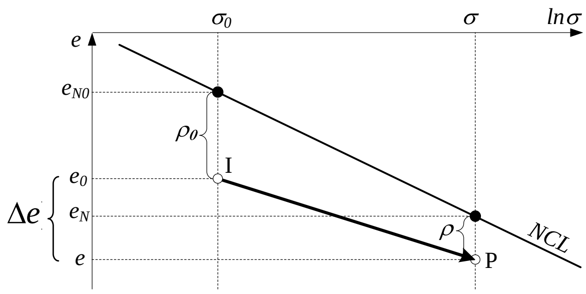

Fig. 2. Change of void ratio in OC clay

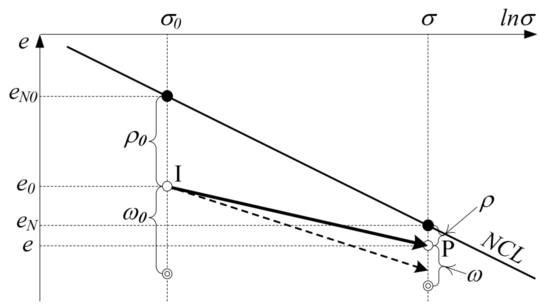

Fig. 3. Change of void ratio in structured clay

$$d(-e) = d(-e)^p + d(-e)^e = \left\{ \frac{\lambda - \kappa}{1 + G(\rho) + Q(\omega)} + \kappa \right\} \frac{d\sigma}{\sigma} \qquad (4)$$

Where, ω is another state variable which represents bonding effect of soils, $Q(\omega)$ is an increasing function which satisfies $Q(0)=0$. The evolution rule of ρ and ω are given by

$$\begin{cases} d\rho = -\{G(\rho) + Q(\omega)\} \cdot d(-e)^p \\ d\omega = -Q(\omega) \cdot d(-e)^p \end{cases} \qquad (5)$$

The loading criteria of this model are presented as follows by assuming the plastic volumetric expansion is zero.

$$\begin{cases} d(-e)^p \neq 0 \ : \ \text{if } d(-e)^p > 0 \\ d(-e)^p = 0 \ : \ \text{if } d(-e)^p \leq 0 \end{cases} \qquad (6)$$

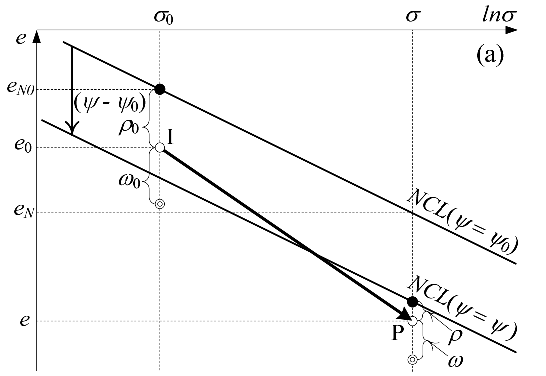

Fig. 4. Change of void ratio considering different effects

Based on the same concept, a general framework in the modeling of one-dimensional case, referring to Fig.4, is given as (reference [7]),

$$d(-e) = d(-e)^p + d(-e)^e = \left\{ \frac{\lambda - \kappa}{1 + G(\rho) + Q(\omega)} + \kappa \right\} \frac{d\sigma}{\sigma} + \frac{d\psi}{1 + G(\rho) + Q(\omega)} \qquad (7)$$

Equation (7) expresses the change of void ratio for different conditions such as the effects of strain rate, temperature, suction, etc. along with the effects of density and bonding in soils. The loading criteria are the same as illustrated in equation (6). Here, ψ is the function of these effect which determines the position of the normally consolidation line (NCL) depending on the particular effect as shown in Fig.4.

The formulation of a model for considering time-dependent behavior will be described a more detail based on the framework shown in equation (7). Fig.5 shows a characteristics curve (e – ln$t$) of soil creep in normally consolidated condition. The void ratio at time t_0 is e_0 and at time t it is e. λ_α is the coefficient of secondary compression. In this interval irreversible plastic change of void ratio occurs. Fig.6 shows the changes of NCL and void ratio for the change of plastic strain from $(-\dot{e})_0^p$ to $(-\dot{e})^p$ when the stress condition moves from the initial state I (σ=σ_0) to the current state P (σ=σ) in the normal consolidation condition. Here, e_0 and e are the initial and current void ratios on the normally consolidation line (NCL) at (ψ=ψ_0) and (ψ=ψ), respectively. Where, ψ is a state variable which changes the position of NCL upward and downward and is responsible for the time effect as described before. The movement of the NCL $(\psi - \psi_0)$ can be obtained from Fig.5, as

$$\psi - \psi_0 = \lambda_\alpha \ln \frac{t}{t_0} = \lambda_\alpha \ln t - \lambda_\alpha \ln t_0 \qquad (8)$$

or,

$$\psi - \psi_0 = \lambda_\alpha \ln \frac{(-\dot{e})_0^p}{(-\dot{e})^p} = \left\{ -\lambda_\alpha \ln(-\dot{e})^p \right\} - \left\{ -\lambda_\alpha \ln(-\dot{e})_0^p \right\} \qquad (9)$$

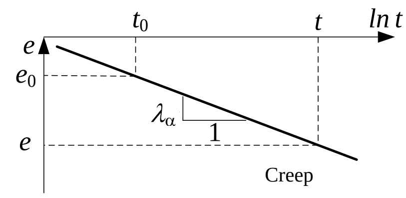

Fig.5. Creep characteristics of normally consolidated soil

The following equation is taken into consideration during creep deformation.

$$(-\dot{e})^p = \frac{d(-\Delta e)^p}{dt} = \frac{d\left(\lambda_\alpha \ln \frac{t}{t_0}\right)}{dt} = \lambda_\alpha \frac{1}{t} \qquad (10)$$

Therefore, from Fig.6 we get,

$$\begin{cases} \psi = \lambda_\alpha \ln t \\ \psi_0 = \lambda_\alpha \ln t_0 \end{cases} \quad \text{or} \quad \begin{cases} \psi = -\lambda_\alpha \ln(-\dot{e})^p \\ \psi_0 = -\lambda_\alpha \ln(-\dot{e})_0^p \end{cases} \qquad (11)$$

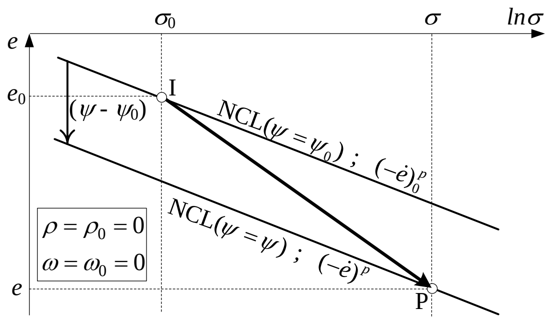

Fig.6. Change of stress and strain rate on NCL

Differentiating equation (10), we obtain

$$\dot{\psi} = \frac{\partial \psi}{\partial t} = \lambda_\alpha \frac{1}{t} = (-\dot{e})^p \qquad (12)$$

Now, assuming equation (11) is valid not only for normally consolidation condition but also for over consolidation and structured soil conditions. Therefore, from equations (7) and (12), we get the total change of void ratio as,

$$d(-e) = \left\{ \frac{\lambda - \kappa}{1 + G(\rho) + Q(\omega)} + \kappa \right\} \frac{d\sigma}{\sigma} + \frac{(-\dot{e})^p . dt}{1 + G(\rho) + Q(\omega)} \qquad (13)$$

or, $$d(-e) \cong \left\{ \frac{\lambda - \kappa}{1 + G(\rho) + Q(\omega)} + \kappa \right\} \frac{d\sigma}{\sigma} + \frac{(-\dot{e})^{p*}}{1 + G(\rho) + Q(\omega)} dt \qquad (14)$$

Where, $(-\dot{e})^{p^*}$ is the plastic component of void ratio at one step immediately before the current step. In this formulation, time variable is not included which makes the model objective. It requires only one additional parameter in the modeling of the characteristics of time effect which is the coefficient of secondary compression λ_α.
Loading criterion, $d(-e)^p > 0$: Loading, : else unloading

3. NUMERICAL SIMULATION

The validity of the proposed model is checked with some simulations of consolidation test (ideal case- one element test) and one-dimensional constant strain rate Oedometer tests. The same parameters of Fujinomori clay which are used in reference [4] are used in these simulations: the compression index λ =0.1040, the swelling index κ=0.010, void ratio on NCL N=0.830 at Pa=98kPa, the parameter for density and confining pressure a =100 and the degradation parameter of bonding b=40 & 100. The evolution rule for ρ is considered as a linear function, $G(\rho)=a\rho$. The evolution rule for ω is also considered as a linear function, $Q(\omega)=b\omega$. The initial rate of the void ratio change is $(-\dot{e})_0^p$ =1.0x10^{-7}. Here, the coefficient of secondary compression λ_α is 0.003.

Fig.7 shows e-lnσ relations for different strain rates in normally consolidated clay under drained condition, simulation results of one element tests. It is seen in the figure that with the increase of strain rates the resistance to compression increases, the lines of constant strain rate are parallel to each other which are commonly seen in the laboratory tests. It is also seen that when strain rate is changed at a point, the curve follows exactly the same path which is supposed to follow. This is valid for both elevating and lowering the strain rates, though in the case of elevating the strain rates it gradually reaches the target curve following the phenomenon of isotachs. Therefore, it can be said that the new model can well produce the strain rate dependency in cohesive soils in normally consolidated condition.

Fig.8 represents e-lnσ relations for different strain rates in over consolidated clay having no structure of the soil. Here, the initial void ratio is 0.73, therefore, the initial value of state variable ρ_0 is 0.1. Fig.9 illustrates the results of over consolidated-structured soil, here, the initial void ratio is 0.73 (ρ_0=0.1), the parameter reflecting initial bonding effect ω_0 is 0.20 and the degradation parameter of bonding b=40. In the figures, the dotted straight line denotes the normal consolidation line for $(-\dot{e})_0^p$ =1.0x10^{-7}. It is found that at the over consolidated state the strain rate dependency is less significant compare to the normally consolidated soil. However, once the soil reaches the ideal NCL the strain rate influences the strength of the soil the same way as the normally consolidated soil, the apparent pre-consolidation stress P_c increases with the increasing rate of strain. Fig.10 shows the results of the over consolidated-structured soil for the same material parameters as illustrated in Fig.9 except for the different degradation parameter of bonding (b=100). The results show both hardening and softening behaviors of soil for different strain rates. The phenomenon of isotachs is seen for both over consolidated and structured soils.

To investigate the consolidation characteristics of naturally deposited clay, simulation of constant strain rate Oedometer test under constant strain loading has been carried out. In the constant strain rate consolidation test, the height of the sample is 2cm as shown in Fig.11(a). In these simulations drainage is allowed at the top boundary of the sample and the bottom boundary is considered as undrained condition. The parameters of the soil are the same as the analyses of the ideal drained condition (one element test). The mean

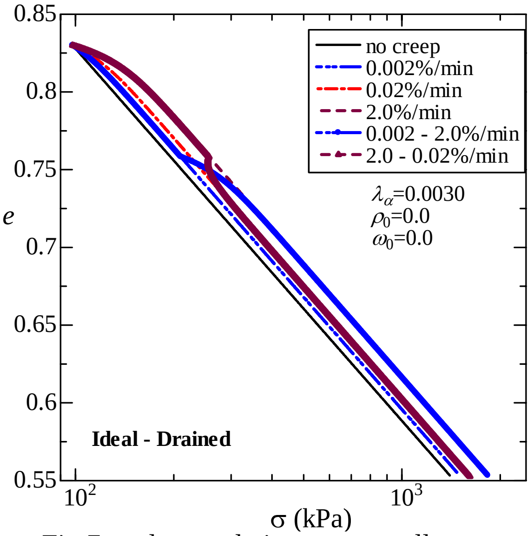

Fig.7 e-lnσ relations: normally consolidated clay

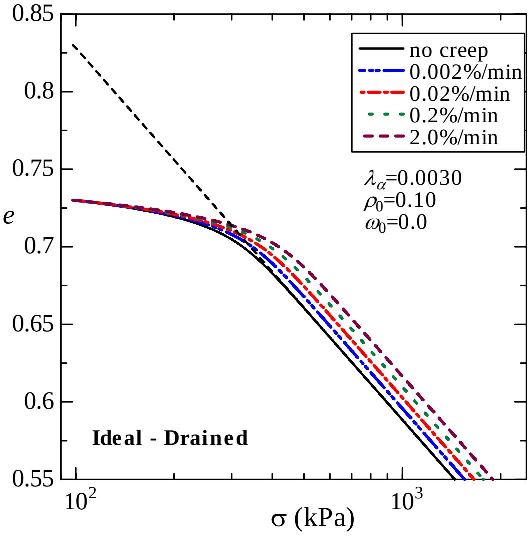

Fig.8 e-lnσ relations: overconsolidated-clay

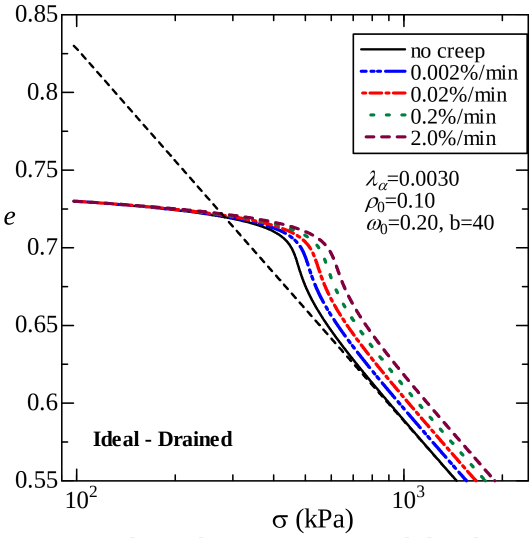

Fig.9 e-lnσ relations: overconsolidated-structured clay effective stress ($\underline{\sigma}'$) is calculated according to equation (15), using the pore water pressure of the bottom most element (u_b), referring to Fig.11(b). This equation is derived assuming a parabolic distribution of the pore water pressure inside the sample.

$$\underline{\sigma}' = \sigma - \frac{2}{3}u_b \qquad (15)$$

For a constant coefficient of consolidation c_v, the following relationship of the coefficient of permeability k with the current void ratio e is used in these simulations.

$$k = k_0 \cdot \exp\left(\frac{e - e_0}{\lambda_k}\right) \qquad (16)$$

Here, at e_0=0.83, k_0=1.0x10^{-5}cm/min and λ_k=0.104.

Fig.12 illustrates the simulation results of the constant strain rate Oedometer test on structured naturally deposited clay. Figure (a) exhibits the results for the degradation parameter of bonding b=40, and figure (b) shows the results for b=100. The vertical axis represents the void ratio as a mass ($\underline{e}$). The abscissas denote the mean effective stress defined with equation (15). The dotted straight line represents the NCL for the reference strain rate ($(-\dot{e})_0^p$ =1.0x10^{-7}), the solid line represents the results of the test where time effect is not considered. It is seen from Figs.9, 10 and 12 that there are no significant differences between the results obtained by the ideal and the constant strain rate Oedometer test. Therefore, it is understood that this type of simulation using the new concept of one-dimensional model can reflects the behavior of consolidation condition of naturally deposited clay.

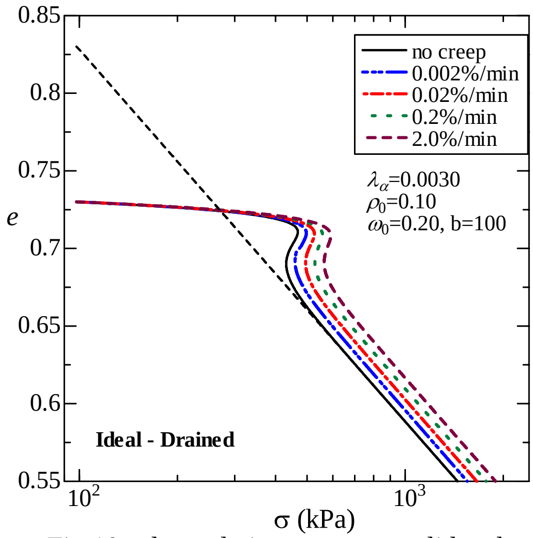

Fig.10 e-lnσ relations: overconsolidated-structured clay

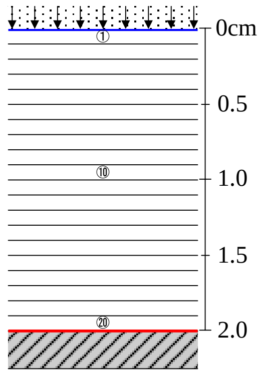

Fig.11(a) FEM mesh

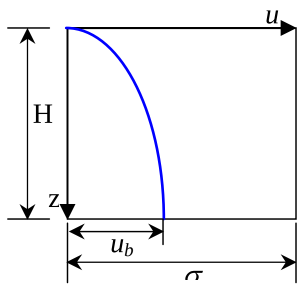

Fig.11(b) Definition of pore water pressure

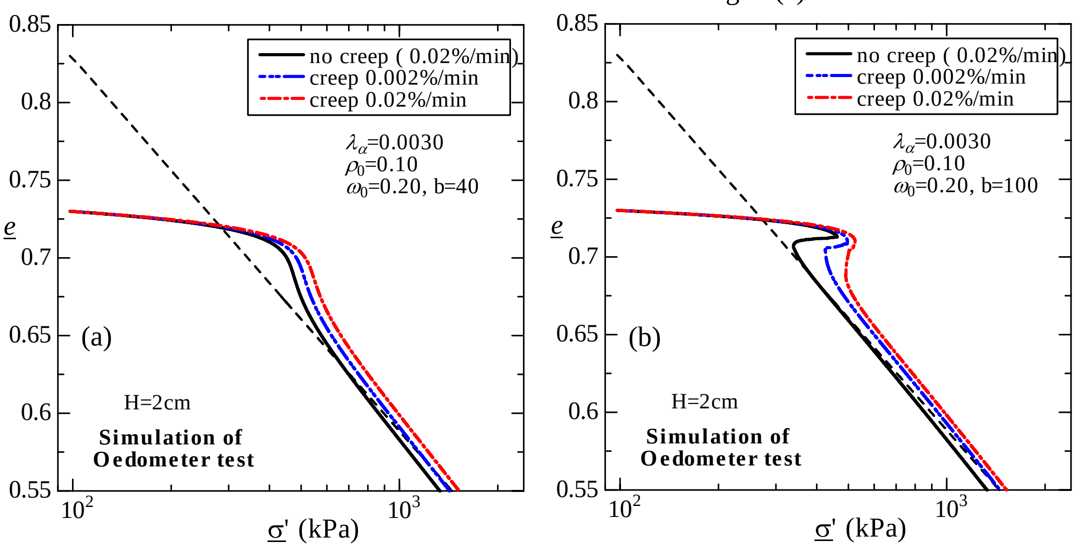

Fig.12 Simulations of Oedometer tests on naturally deposited clay

Fig.13 shows e-ln$t$ response of conventional oedometer tests for normally consolidated clay, where the initial stress is 98kPa and the sudden increment of stress is 98kPa. After applying the stress increment the consolidation behavior of soil is investigated for the different values of the coefficient of secondary compression (λ_α). The vertical axis represents the void ratio as a mass. The solid curve represents the results where the time effect is not considered. Figure (a) indicates the results for the sample height (H) of 1cm and figure (b) for the height of 5cm. It is seen that the delayed consolidation occurs when the time effect is considered, which shows creep behavior of soil. The larger the value of λ_α the more the delay in consolidation is observed. For considering the time-dependent behavior of soil, the void ratio (settlement)- logarithm of time curve forms the shape of reverse ‘s’ regardless the height of the sample which is commonly seen in the literature related to creep settlement. During secondary consolidation the slopes of the curves are the same as the coefficient of secondary compression which are employed in the simulations.

Fig.14 represents creep characteristics of normally consolidated clay for different heights of the sample. Here, the heights of the samples are 1cm, 5cm and 10cm. The coefficient of secondary compression (λ_α) is 0.003 in figure (a) and it is 0.045

for figure (b). Here, the initial stress of each sample is 98kPa and the increment of the stress is 98kPa. These figures describe the effects of sample height. It is found that after passing some time from the application of the load, the consolidation curves for different sample height converse to a single curve. This is valid for all values of the coefficient of secondary compression. Fig.15 illustrates creep characteristics of normally consolidation clay for different stress increments. Here, the height of the sample is 5cm and the coefficient of secondary compression (λ_α) is 0.003. It is seen from the figure that though the settlement of the sample is large for the larger value of stress increment, the final slope of each curve is the same which is equal to the coefficient of secondary compression. Therefore, it can be said that the final slope of the consolidation curve does not depend on the amount of applied load. It is also noticed that the primary consolidation takes longer time for the larger value of stress increment.

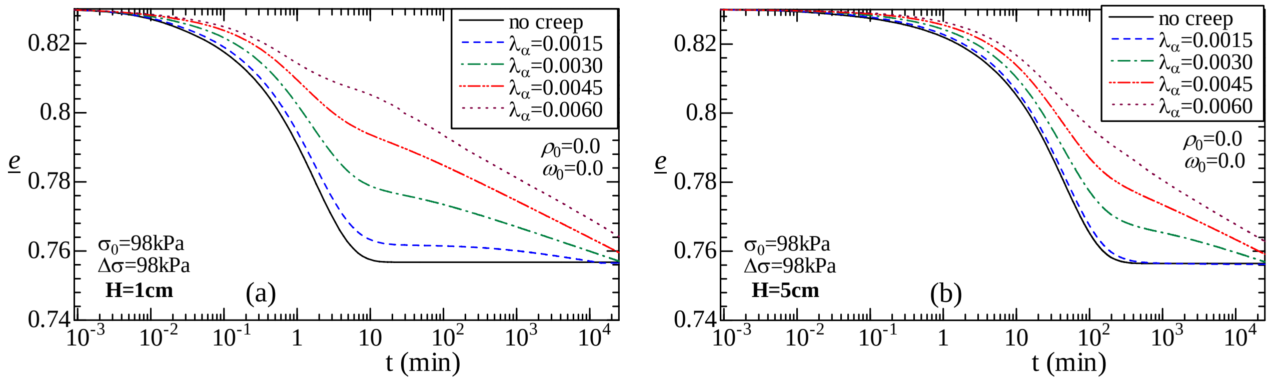

Fig.13 Creep characteristics of normally consolidated clay for different values of λ_α

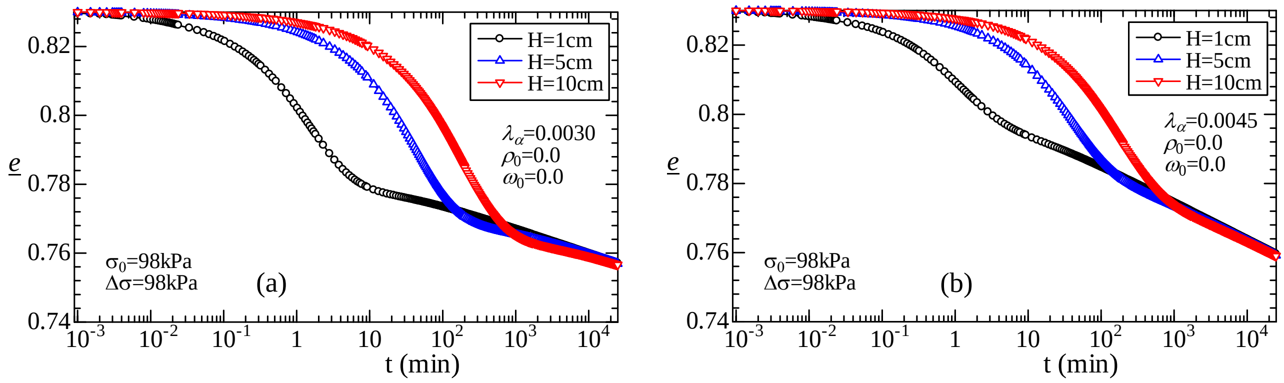

Fig.14 Creep characteristics of normally consolidated clay for different heights of the samples

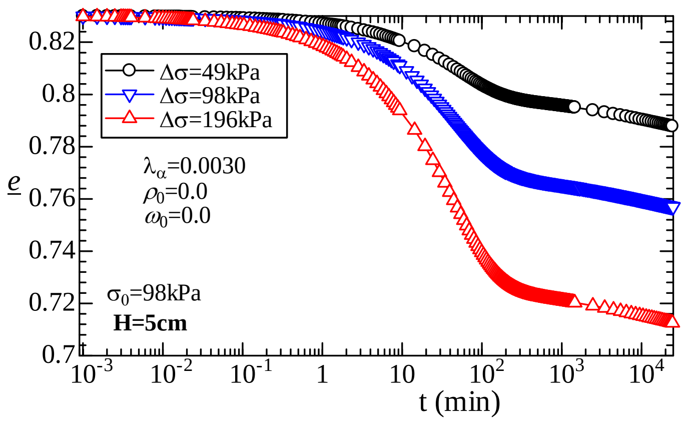

Fig.15 Creep characteristics of normally consolidated clay for different stress increments

Fig.16 shows creep characteristics of over consolidated clay for different values of the coefficient of secondary compression. Here, the over consolidation ratio is 1.70 and the height of the sample is 5cm. The application of stress increment is 98kPa. In figure (a) the time is in logarithmic scale, and it is in normal scale in figure (b). It seems that the results of the over consolidated clay is different and there is an acceleration of settlement at some where on the way compare to the results of the normally consolidated clay (Figs.13(b) & 16(a)) where the time is represented in logarithmic scale, however, this is not seen in Fig.16(b) where the time is represented in normal scale. For the over consolidated clay the delayed consolidation occurs the same way as the normally consolidated clay when the time effect is considered.

Fig.17 illustrates creep characteristics of over consolidated clay for different over consolidation ratio (OCR). Here, four values of OCR (1.0, 1.70, 2.00 and 2.90) are considered. The coefficient of secondary compression (λ_α) is 0.003, and the height of the sample is 5cm. The application of stress increment is 196kPa, double the initial state of stress. The solid line represents the result of normally consolidated clay (OCR=1.0). Figure (a) shows *e*-ln*t* relation and figure (b) represents ψ–ln*t* relation. As mentioned in the section of model formulation, ψ is a state variable which represents the movement of NCL. For the normally consolidated clay ψ decreases linearly with ln*t*, on the other hand, for the over consolidated clay the change of ψ is not linear with logarithm of time. However, once the curve of over consolidated clay reaches the curve of the normally consolidated clay, ψ decreases linearly with ln*t*.

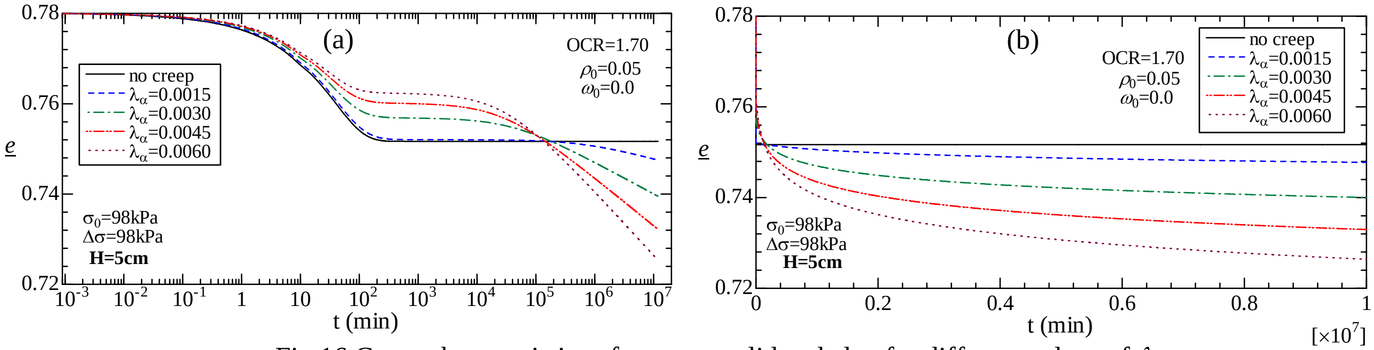

Fig.16 Creep characteristics of over consolidated clay for different values of λ_α

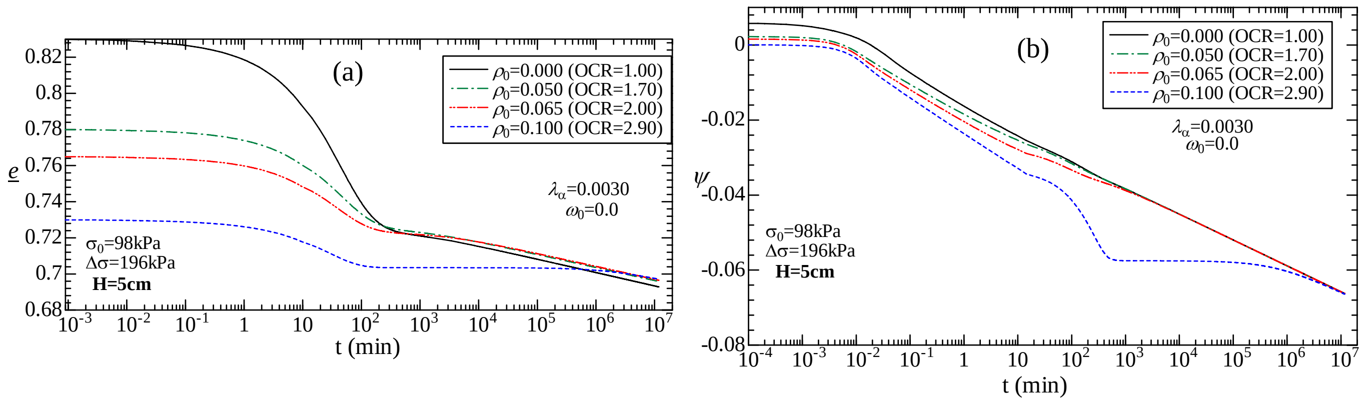

Fig.17 Creep characteristics of over consolidated clay for different over consolidation ratio

CONCLUSION

A rational modeling for considering the characteristics of time effect has been presented. The validities of the model have been checked by the simulations of one-dimensional consolidation on normally and overconsolidated structured soils. The model can explain well the behavior of secondary consolidation of naturally consolidated soil, over consolidated soil and structured soil

REFERENCE

1) Asaoka, A., Nakano, M. Noda, T and Kaneda, K. (2000): Delayed compression/consolidation of natural clay due to degradation of soil structure, *Soils & Foundation*, vol. 40(3): 75-85.
2) Sekiguchi. H. (1985): Macrometric approaches –static– intrinsically time–dependent, *Constitutive Laws of Soils-Proc. of 11th ICSMFE,* San Francisco, 66-98.
3) Hashiguchi, K. and Okayasu. T. (2000): Time-dependent elastoplastic constitutive equation based on the subloadingsurface model and its application to soils, Soils and Foundations, 40(4), pp19-36.
4) Nakai, T., Hinokio, M. (2004): A simple elastoplastic model for normally and over consolidated soils with unified material parameters, *Soils & Foundation*, vol. 44(2): 53-70.
5) Nakai, T (2008): Description of Elastoplastic Behavior of Soil as a One-dimensional Model, Magazine of Japanese Geotechnical Society, vol.56(9), pp.6-9.
6) Nakai, T, Zhang, F, Kyokawa, H, Kikumoto, M & Shahin, H.M. (2008): Modeling the influence of density and bonding on geomaterials, *Proc. of 2nd International Symposium on Geotechnics of Soft Soils,* Keynote Paper, Glasgow, 65-76.
7) Nakai, T, Shahin, H.M., Kikumoto, M, Kyokawa, H & Zhang, F (2009): Simple and unified method for describing various characteristics of geomaterials – Influences of density, bonding, time effects and others, *Journal of Applied Mechanics JSCE*, vol.12, (in press).