APPLICATION OF AN EXPERT SYSTEM TO CONSTITUTIVE MODEL DETERMINATION AND INVERSE ANALYSIS PARAMETERS IDENTIFICATION METHOD

Piotr DZIADZIUSZKO[1,2] and Yasuaki ICHIKAWA[1] and Zbigniew SIKORA[2] and Takafumi SEIKI[1] and Yong Seok SEO[1]

[1] Graduate School of Geotechnical and Environmental Engineering, Nagoya University, Chikusa, Nagoya, 464-01, Japan
[2] Geotechnical Department, Environmental Engineering Faculty, Technical University of Gdańsk, Narutowicza 11, 80-952 Gdańsk, Poland

ABSTRACT: New concept of an *Expert System* application as decision support tool for constitutive models determination and Inverse Analysis material parameters identification procedure in the field of Geotechnics are presented in this paper. The introduction to Expert Systems as a useful tool in the situations requiring the matter of fact expertise is provided with insist on description of their characteristics and advantages. The new concept of material parameters identification procedure called *Dual Boundary Control Method* as a variety of *Inverse Analysis Method* in its general form is further presented. This general description may be treated as the theoretical base for application of *Dual Boundary Control Method* to various classes of constitutive models for soil, ranging from linear elastic to hypoplastic ones. As an example of its application the ***elasto-plastic*** model (with assumed Drucker-Prager yield criterion) parameters identification is presented. Finally authors summarize the advantages of unique combination of an *Expert System* and *Inverse Analysis Method* as a step towards precision and completeness in the field of constitutive modeling of geotechnical materials.

KEYWORDS: *Expert System, Constitutive Model, Inverse Analysis, Dual Boundary Control Method, Material Parameters, Numerical Simulation*

Introduction

Recently Expert Systems gained big popularity in many fields of human activity, where the decision-making process must be supported by matter-of-fact expertise. In fact so far there were only a few attempts to apply the Expert Systems in the field of Geotechnics. Proposed Expert System is in these circumstances a new approach to the constitutive modeling of geotechnical materials.

Determination of constitutive model that by now are limited to *linear elastic, elasto-plastic, visco-plastic, and hypoplastic* is both the first stage of expertize acquisition and the first target of its application.

The second stage and the second target is material parameters identification for the constitutive model determined in the previous stage. This is obtained by means of very promising *Dual Boundary Control Method* which is an original concept of Ichikawa. This concept is presented in the general form providing a theoretical base for its application to the variety of constitutive models of soil materials.

Introduction To Expert Systems

An Expert System (ES) may be defined as sophisticated computer technology that provides an alternative to a human expert in some specific area of interest. Working with ES reminds the dialog with intelligent device, thus Expert Systems are classified as the part of an *Artificial Intelligence* (AI).

Expert Systems have been successfully employed in various domains of human activity such as medical diagnostics, nuclear power-plants control, geological surveys and many others. Due to functionality they may be divided into three main categories

- *advisory* - systems which produce expertise that can be verified and rejected by user. The examples of this kind of systems are ones for supporting financial and banking decisions,
- *dictatorial* - decision making process performed by these kind of system is basically out of human ability. This category involves the systems created to solve very complicated problems - too large for human control or where the access of human expert is difficult e.g. nuclear power plants control, and
- *verifying* - systems which deal with input and output proposed by human simultaneously. These expert systems analyze the process of decision making and verify its correctness. This kind of systems are for instance applied to verify environmental-management decisions.

Expert systems consist of the following main components [8]

- *knowledge base* (KB) - place where the knowledge in a form of implemented facts is stored,

- *data base* - place where additional data required by the system is stored,
- *inference engine* - inference procedures consisting of logical rules implemented in KB,
- *explanation engine* - explanation procedures activated in different stages of dialog,
- *dialog-control engine* - part of an ES responsible for communication between an user and the system often including procedures verifying data acquired during the consultation and correctness of system's inference,
- *data acquisition engine* - procedures which allow KB modifications.

ES-construction may be divided into three main stages

- *knowledge acquisition*,
- *knowledge structuralisation*, and
- *knowledge transformation.*

The most time consuming task during ES construction is *knowledge acquisition* which is usually done by means of interviews with human experts, referring to literature covering specific subject and by means of professional experience of *knowledge engineer*[1]. It is worth noting that nowadays Internet Web became precious source of information in many areas of interest.

Properly designed Expert Systems are characterized by *modular* and *open structure* represented by a system of modules which can be independently updated.

Due to functionality expert systems can be divided into following categories:

- *diagnostic* - the expertise is obtained by means of existing data,
- *prognostic* - the expertise is in the form of prediction of the future state,
- *planistic* - the expertise is in form of the description of an arbitrary state treated as a target and the way of reaching it.

During all stages of Expert System construction a knowledge engineer must take into consideration following functionality criteria characterizing properly designed and constructed ES

- *correctness of the system* - ensuring high level of expertise obtained in required time,
- *flexibility* - indicating the ability of the system to solve wide brand of problems in a certain domain,
- *complexity* - degree of naturally determined complication corresponding to the field where an ES is applied.

[1]The person responsible for ES construction

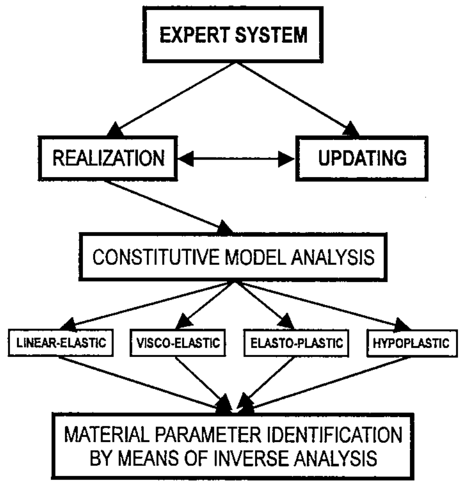

Figure 1: The structure of SIAES

Soil Material Parameters Inverse Analysis Expert System

The Expert System for constitutive models and their material parameters identification is currently developed. Its name **SIAES** stands for **S**oil Material Parameters **I**nverse **A**nalysis **E**xpert **S**ystem.

Due to classifications given in the previous sections described ES can be classified as the advisory and diagnostic, knowledge-based system.

SIAES is planned to be constructed in ESTA that is stand-alone environment for constructing advisory and decision support systems developed in PDC Visual Prolog - logical programming language.

The main purpose of this ES is determination of constitutive model properly describing behaviour of given soil material. This is done by means of the *consultation*[2] during which facts, data implemented in the system's knowledge base and inference engine are used to produce an expertise. During consultation some information in form of macroscopic characteristic of given soil material and results of *in situ* measurements are requested by the system. Moreover because of the big variety of constitutive models of soil material some theoretical knowledge of constitutive modeling is also requested from an user.

The knowledge base of SIAES is planned to be of a modular and open structure. Because of the limited SIAES developing time only some knowledge base modules corresponding to constitutive models of soil materials are currently planned to be equipped. These models are: *linear elastic*, *visco-elastic*, *elasto-plastic* and *hypoplastic.* The block scheme of the SIES structure including knowledge base modules corresponding to above mentioned constitutive models is presented in Fig. 1

The second stage of consultation with SIAES is identification of material parameters involved in the constitutive model determined in the first stage. This is obtained by means of system's calls to external programs

[2]Dialog with ES which leads to expertise

employing Inverse Analysis identification method that will be described in the proceeding section.

Summarizing, the final results obtained after consultation session with SIAES are:

- the choice of proper constitutive model of soil corresponding to the given soil material with so called reasoning path rapport,
- values of material parameters involved in the previously chosen model obtained by means of an inverse analysis method, and
- information of the inverse analysis process in a form of charts representing e.g. convergence of the proposed optimization technique.

Generalized Procedure of Material Parameter Identification by Means of *Dual Boundary Control Method*

The inverse analysis applied to the material parameter identification problem is a prediction procedure to determine parameters of the system involving the effect of the material property and the boundary conditions [6].

In geotechnical engineering this method was introduced by Kavanagh [7] and it may be classified into the inverse formulation [1], [9] and the direct formulation [2].

In SIAES a direct formulation is used in the form of so called the *Dual Boundary Control Method* that is an original concept introduced by Ichikawa [6]. The core of this method is to introduce "control" or "observational" boundary conditions (which indeed consist of observational data measured on the part of the boundary) into the "direct" or "given" boundary conditions, then desired material parameters are calculated by means of an optimization technique. Note that in the system of equilibrium the "direct" boundary conditions are of two types:

1. the displacement boundary condition, and
2. the traction boundary condition,

therefore we have also two corresponding types of the "observational" boundary conditions.

Generalized procedure of material parameters identification by means of *Dual Boundary Control Method* may be summarized as follows:

For a given load on the traction boundary, let a displacement be observed on a part of the same traction boundary, then material properties are determined under the given traction condition to fit the observed displacement data by means of optimization technique. The "control" displacement boundary condition is understood here as a constrained condition. A numerical part of presented procedure is based on a two-stage finite element approximation for both equilibrium and constitutive equations. The variety of this method (with one stage FE approximation) was successfully employed in the case of a linear elastic material [6] and for a damage mechanics problem [10].

The incremental formulation of the problem for a nonlinear material may be given as follows:

Equation of equilibrium

$$\nabla \cdot d\boldsymbol{\sigma} = 0 \qquad \text{in } \Omega, \tag{1}$$

Constitutive law

$$d\boldsymbol{\sigma} = \boldsymbol{D} d\boldsymbol{\varepsilon}, \tag{2}$$

"Direct" or "given" boundary conditions

$$d\boldsymbol{u} = d\hat{\boldsymbol{u}} \quad \text{on } \partial\Omega_u \quad \text{(displacement boundary)}, \tag{3}$$

$$d\boldsymbol{\sigma}\boldsymbol{n} = d\hat{\boldsymbol{t}} \quad \text{on } \partial\Omega_t \quad \text{(traction boundary)}. \tag{4}$$

where $d\boldsymbol{\sigma}$ is the stress increment given in the region Ω, with boundary $\partial\Omega$. $\boldsymbol{n}$ is the outward unit normal vector, and $\boldsymbol{D}$ the fourth order tensor of material constants depending on the chosen constitutive model.

Observed data are given in the form of a displacement increment $d\bar{\boldsymbol{u}}$ on $\partial\Omega_{t'} \subset \partial\Omega_t$, and/or a traction increment $d\bar{\boldsymbol{t}}$ on $\partial\Omega_{u'} \subset \partial\Omega_u$. They are referred to as

"Observational" or "control" boundary conditions

$$d\boldsymbol{u} = d\bar{\boldsymbol{u}} \qquad \text{on } \partial\Omega_{t'} \subset \partial\Omega_t \tag{5}$$

in case of displacements, and

$$d\boldsymbol{\sigma}\boldsymbol{n} = d\bar{\boldsymbol{t}} \qquad \text{on } \partial\Omega_{u'} \subset \partial\Omega_u \tag{6}$$

in case of tractions.

The virtual work equation corresponding to the system (1), (3) and (4) can be written as

$$\int_{\partial\Omega_t} d\hat{\boldsymbol{t}} \cdot \delta(d\boldsymbol{u}) dS - \int_{\Omega} d\boldsymbol{\sigma} \cdot \delta(d\boldsymbol{\varepsilon}) dV = 0 \tag{7}$$

for an arbitrary virtual displacement $\delta(d\boldsymbol{u})$. It is further assumed that $\delta(d\boldsymbol{u}) = \boldsymbol{0}$ on $\partial\Omega_u$.

The constitutive relation (2) is used, and the finite element discretization procedure for (7) results in the following algebraic equation:

$$\boldsymbol{K} d\boldsymbol{U} = d\boldsymbol{F} \tag{8}$$

$$\boldsymbol{K} = \int_{\Omega} \boldsymbol{B}^t \boldsymbol{D} \boldsymbol{B} dV \quad d\boldsymbol{F} = \int_{\partial\Omega_t} \boldsymbol{N}^t d\hat{\boldsymbol{t}} dS$$

where $\boldsymbol{N}$ is the matrix of shape functions, and $\boldsymbol{B}$ the strain-displacement matrix. The approximated solution $d\boldsymbol{u}_h$ is then written as

$$d\boldsymbol{u}_h \simeq d\boldsymbol{u} = \boldsymbol{N} d\boldsymbol{U}$$

The observational boundary conditions (5) and (6) are directly discretized as

$$\boldsymbol{S}_u d\boldsymbol{U} = d\bar{\boldsymbol{U}} \tag{9}$$

in case of displacements, and

$$S_t dF = \int_{\partial\Omega_u} N^t d\bar{t} dS = d\bar{F} \tag{10}$$

in case of tractions, where S_u and S_t are diagonal matrices for choosing observational boundary nodes of displacement and traction, respectively. That is, if i-th node is the displacement observational boundary, and if j-th node is the traction one, we have

$$\begin{array}{cc} S_u = \begin{bmatrix} 0 & & & & \\ & \ddots & & \mathbf{0} & \\ & & 1 & & \\ & \mathbf{0} & & \ddots & \\ & & & & 0 \end{bmatrix} & \cdots i\text{-th row} \\ \begin{array}{c} \vdots \\ i\text{-th column} \end{array} & \end{array}$$

$$\begin{array}{cc} S_t = \begin{bmatrix} 0 & & & & \\ & \ddots & & \mathbf{0} & \\ & & 1 & & \\ & \mathbf{0} & & \ddots & \\ & & & & 0 \end{bmatrix} & \cdots j\text{-th row} \\ \begin{array}{c} \vdots \\ j\text{-th column} \end{array} & \end{array}$$

Elasto-plastic material parameters P (written in the vector form) which are to be identified are nested in the stiffness matrix K, and consequently in the material constants matrix D what is written

$$D = D(P).$$

For identifying P, we apply the Newton's iteration scheme as the variety of optimization technique into (8), (9) and (10). That is, in the k-th iteration step, we have

$$K(P^k)\Delta(dU)^k + (\frac{\partial K}{\partial P})^k \Delta P^k dU^k - \Delta(dF)^k = [dF - KdU]^k \tag{11}$$

$$S_u \Delta(dU)^k = d\bar{U} - S_u dU^k \tag{12}$$

$$S_t \Delta(dF)^k = d\bar{F} - S_t dF^k \tag{13}$$

where

$$\Delta(dU)^k = dU^{k+1} - dU^k,$$
$$\Delta(dF)^k = dF^{k+1} - dF^k,$$
$$\Delta P^k = P^{k+1} - P^k.$$

Representing (11), (12) and (13) in the matrix form yields

$$G\Delta X = R \tag{14}$$

where

$$G = \begin{bmatrix} K(P^k) & -I & (\frac{\partial K}{\partial P}\Delta U)^k \\ S_u & 0 & 0 \\ 0 & S_t & 0 \end{bmatrix}$$

$$\Delta X = \begin{bmatrix} \Delta(dU)^k \\ \Delta(dF)^k \\ \Delta P^k \end{bmatrix} = \begin{bmatrix} dU^{k+1} - dU^k \\ dF^{k+1} - dF^k \\ P^{k+1} - P^k \end{bmatrix}$$

$$R = \begin{bmatrix} dF - K(P^k)dU^k \\ d\bar{U} - S_u(dU)^k \\ d\bar{F} - S_t(dF)^k \end{bmatrix}$$

Since the number of observed data (boundary displacements and/or tractions increments) may exceed the number of unknowns, (14) may be an over-determined system. Thus, we need to introduce the least square method with the error function defined as

$$\mathcal{E} = \frac{1}{2}(GdX - R)(GdX - R) \tag{15}$$

The condition $\delta\mathcal{E} = 0$ implies

$$G^T G dX = G^T R \tag{16}$$

We should also note that K, S_u and S_t are symmetric, and $S_u^2 = S_u$, $S_t^2 = S_t$, $S_t dU = d\bar{U}$ and $S_t dF = d\bar{F}$ which drastically simplifies the formulation and shorten further calculation time. Material parameters P can now be identified by means of the iteration scheme (16).

Example of Application of *Dual Boundary Control Method* for Elasto-Plastic Soil Parameters Identification

As an example we here present application of the *Dual Boundary Control Method* to *elasto-plastic* soil material parameters identification. The *Drucker-Prager* yield criterion including dilatancy characteristic is applied and it is also assumed that soil material treated here is *hydrostatically symmetric* i.e. variables of the response function are only the mean and deviatoric components of stress and strain [4].

In described example parameters that are to be identified are the discrete values of isotropic hardening function with assumed strain hardening[3] corresponding to given load increments. After Inverse Analysis identification procedure they are compared with the values of the theoretical hardening function of the form

[3]It is assumed here that *softening phenomenon* is disregarded.

$$K = K_0 + \sum_i a_i\{1 - \exp(-e^p/\tau_i)\} + \sum_i b_i\{1 - \exp(-\bar{\varepsilon}^p/\omega_i)\} + \sum_i c_i[\{1 - \exp(-e^p/\tau_i)\}\{1 - \exp(-\bar{\varepsilon}^p/\omega_i)\}] \quad (17)$$

where e^p is the deviatoric plastic strain, $\bar{\varepsilon}^p$ the mean plastic strain, and τ_i and ω_i are the spectral values obtained by means of the *Laplace transformation* theory. For details, see Ichikawa [3].

The Drucker-Prager yield function is written

$$f(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}^p) = \alpha\bar{\sigma} + s - K(e^p, \bar{\varepsilon}^p) \quad (18)$$

where $\boldsymbol{\varepsilon}^p$ is the effective plastic strain, $\boldsymbol{\sigma}$ is the deviatoric stress, $\bar{\sigma}$ the mean stress, and α the material constant.

If the plastic strain increment $d\boldsymbol{\varepsilon}^p$ is coaxial with the stress $\boldsymbol{\sigma}$ we can introduce so called *dilatancy factor* defined by

$$\beta = \frac{d\bar{\varepsilon}^p}{de^p} = \frac{\partial q/\partial\bar{\sigma}}{\partial q/\partial s} \quad (19)$$

where

$$d\boldsymbol{e}^p = \lambda\frac{\partial q}{\partial s}\boldsymbol{m} \quad (\boldsymbol{m} = \boldsymbol{s}/s)$$

$$d\bar{\boldsymbol{\varepsilon}}^p = \lambda\frac{\partial q}{\partial\bar{\sigma}}\boldsymbol{n} \quad (\boldsymbol{n} = \bar{\boldsymbol{\sigma}}/\bar{\sigma})$$

$d\boldsymbol{e}^p$ is the deviator of plastic strains increments, $d\bar{\boldsymbol{\varepsilon}}^p$ is the tensor of mean plastic strains increments, de^p is the norm of plastic strains increments deviator, $d\bar{\varepsilon}^p$ is the mean plastic strain increment, $\boldsymbol{s}$ is the stress deviator, and q is so called *plastic potential function* which is replaced by yield function (18) for assumed *associated flow rule*.

Henceforth after FE discretization of functions (21) and (23) we obtain so called *constitutive elements*[4] of the form:

$$K(e^p, \bar{\varepsilon}^p) \simeq \sum_i K^i\phi^i(e^p, \bar{\varepsilon}^p)$$

$$\beta(\bar{\sigma}) \simeq \sum_i \beta^i\psi^i(\bar{\sigma}) \quad (20)$$

where $\phi^i(e^p, \bar{\varepsilon}^p)$ and $\psi^i(\bar{\sigma})$ are interpolation functions in the spaces $(e^p, \bar{\varepsilon}^p)$ and $(\bar{\sigma})$, respectively.

The elasto-plastic constitutive equation is then given by

$$d\boldsymbol{\sigma} = \boldsymbol{D}^{ep}d\boldsymbol{\varepsilon} \quad (21)$$

where

$$\boldsymbol{D}^{ep} = \boldsymbol{D}^e - \frac{\boldsymbol{D}^e(\boldsymbol{m} + \sum\beta^i\psi^i\boldsymbol{n}) \otimes \boldsymbol{D}^e(\boldsymbol{m} + \alpha\boldsymbol{n})}{h' + (\boldsymbol{m} + \alpha\boldsymbol{n}) \cdot \boldsymbol{D}^e(\boldsymbol{m} + \sum\beta^j\psi^j\boldsymbol{n})} \quad (22)$$

[4]The Finite elements used in approximating the equation of equilibrium are called the *structural elements*

where

$$h' = \sum K^i(\frac{\partial\phi^i}{\partial e^p} + \sum\beta^j\psi^j\frac{\partial\phi^i}{\partial\bar{\varepsilon}^p}).$$

is a derivative of hardening function.

The comprehensive procedure of identifying material parameters by means of the *Dual Boundary Control Method* may be divided into following stages [5]:

1. Identification of elastic constants involved in matrix of elastic constants $\boldsymbol{D}^e$ by means of inverse analysis method for linear elastic problems proposed by Ichikawa and Ohkami [6].
2. Determination of initial yielding constants α and K_0 by means of stress distributions of initial yielding under several confining pressures in triaxal tests.
3. Identification of the dilatancy parameter β^i and nodal values of hardening function for the load increments $d\boldsymbol{F}$ by means of the iteration scheme (16).
4. Repeating step 3 until assumed accuracy condition e.g.

   $$\boldsymbol{X}^T\boldsymbol{X} < \varepsilon^2 \quad \text{for} \quad 0 < \varepsilon \ll 1 \quad (23)$$

   is satisfied.

To show the efficiency of proposed method we give an example of numerical simulation of triaxial test. In this case the material for which discrete values of hardening function are to be identified is Oya tuff. The lateral displacement measured at three points by means of O-ring type gauges is applied as the control boundary data. Stresses at the lateral surface are set to be zero, and stresses at the top surface are confined.

Comparison between results computed by the proposed method and theoretical hardening function (17) is presented in Fig. 2 indicating good agreement.

The parameters of Oya tuff are as follows:

Young's modulus $E = 2000$ MPa
Poisson's ratio $\nu = 0.12$
Initial yield surface parameters $\alpha = 0.29, K_0 = 5.8$ MPa
Dilatancy factor $\beta = 0.4$ for $\sigma_3 = 0.0$

Parameters of hardening function (17):

$\tau_1 = 5.560 \times 10^{-4}$, $\omega_1 = 0.331 \times 10^{-4}$,
$\tau_2 = 1.301 \times 10^{-3}$, $\omega_2 = 1.987 \times 10^{-4}$,
$a_1 = 8.198$ MPa, $a_2 = -3.551$ MPa,
$b_1 = 1.442$ MPa, $b_2 = -6.690$ MPa,
$c_1 = -1.541$ MPa, $c_2 = 7.437$ MPa.

Conclusions

In this paper we presented a new approach to constitutive model determination and soil parameter identification by means of combination of the Expert System with the *Dual Boundary Control Method* as the variety of Inverse Analysis Method.

An introduction to Expert Systems with regard to designed ES features and construction stages is also included.

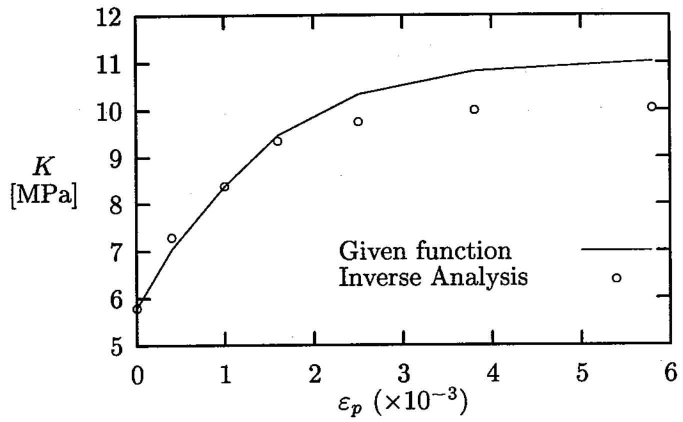

Figure 2: Hardening function values computed by proposed inverse analysis method and given by the equation (17)

Actually developed Expert System called SIAES and its advantages are also described with insist on the expected results obtained after its consultation.

The general procedure of *Dual Boundary Control Method* is also presented in the form applicable to material parameters identification of the variety of constitutive models of soil materials.

Finally an example of application of the parameters identification method based on the *Dual Boundary Control Concept* is given for the elasto-plastic constitutive model with Drucker-Prager yield criterion. In this case numerically simulated conventional triaxial test of soil material indicate good agreement with theoretical estimation.

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